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TWO-DIMENSIONAL SYSTEMS: A METHOD FOR THE DETERMINATION OF THE FINE STRUCTURE CONSTANT

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The energy spectrum of a two-dimensional electron gas (2DEG) becomes fully quantized into Landau levels, if a strong magnetic field is applied perpendicular to the plane of the 2DEG. Under experimental conditions where an integer number i of Landau levels is occupied, the Hall resistance (Hall voltage devided by the current through the 2DEG) becomes $R_H = h/e^2 i$ (h = Planck constant, e = elementary charge). Our experiments demonstrate that the value of this new "Quantum Resistor" is within our experimental uncertainty of about 10^{-6} independent of the substrate material (silicon MOSFETs or GaAs–$Al_xGa_{1-x}As$ heterostructures), the lateral dimensions of the 2DEG, and the strength of the magnetic field or other parameters. Since the fine structure constant is determined by the ratio h/e^2, measurements of the quantized Hall resistance $R_H = h/e^2 i$ allow determining directly the fine structure constant.

1. Introduction

The fine structure constant α is one of the most important fundamental physical constants, since this dimensionless constant, which combines the Planck constant h with the elementary charge e and the velocity of light c, characterizes the interaction of charged particles with electromagnetic fields. In SI units α is given by the expression (permeability of the vacuum $\mu_0 = 4\pi \times 10^{-7}$ H/m, $c = 2.99792458 \times 10^8$ m/s)

$$\alpha = (e^2/h)\mu_0 c/2. \tag{1}$$

The small value of $\alpha \approx 1/137$ allows one to use α as an expansion parameter in quantum electrodynamics (QED) and the corresponding theories for the anomalous magnetic moment of the electron (a_e) and the muonium hyperfine splitting (Mhfs) are developed to a high accuracy. The QED theory can explain highly accurate measurements of a_e and Mhfs, if the following values for the fine structure constant are assumed [1]:

$$\alpha^{-1}(a_e) = 137.035987 \pm 0.000014, \tag{2}$$

$$\alpha^{-1}(\text{Mhfs}) = 137.035989 \pm 0.000047. \tag{3}$$

High precision determinations of the fine structure constant without using

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QED theory are based on eq. (1) which can be rewritten in the form [2]:

$$\alpha^{-1} = (C_1/\gamma_p)^{1/2}, \tag{4}$$

where C_1 is a combination of fundamental constants which is known with an uncertainty much smaller than the uncertainty of the gyromagnetic ratio of the proton γ_p. Williams and Olsen got from their γ_p measurements an α value of [2]

$$\alpha^{-1}(\gamma_p) = 137.035963 \pm 0.000015, \tag{5}$$

but there is still no agreement, even at the ppm level, between equivalent experimental results obtained at different laboratories. The officially recommended value for α is [3]

$$\alpha^{-1} = 137.03604 \pm 0.00011. \tag{6}$$

In order to check the QED theory (eqs. (2) and (3)) an accurate value for the fine structure constant using eq. (1) is necessary. From Hall effect measurements on two-dimensional systems (quantized Hall resistance R_H), a direct determination of h/e^2 and therefore α (eq. (1)) seems to be possible [4].

At the Second International Conference on Precision Measurement and Fundamental Constants (June 1981) already three different groups presented values for the fine structure constant using the quantized Hall resistance [5–7], and the present values are

$$\alpha^{-1} = 137.035968 \pm 0.000023 \quad \text{(USA)},$$

$$\alpha^{-1} = 137.03589 \pm 0.00012 \quad \text{(Japan)},$$

$$\alpha^{-1} = 137.03592 \pm 0.00008 \quad \text{(Germany)}.$$

A reduction in the uncertainty down to a level of 10^{-7} is expected in the near future using cryogenic current comparators [5,8]. At this level of uncertainty a test of QED theory should be possible.

One further aspect of interest in the quantized Hall resistance is the fact that this "quantum resistor" has values which depend exclusively on fundamental constants. Therefore, similar to the use of the Josephson effect in connection with the Volt, the quantized Hall resistance may be used as a resistance standard.

Such applications of the quantized Hall resistance are only possible, if all corrections are known. Up to now, the experimental verification of a parameter-independent value for the quantized Hall resistance justifies the assumption that R_H depends only on fundamental constants. In order to discuss possible corrections, experimental data and theories related to the quantized Hall resistance are summarized in the following sections.

2. Quantum transport of a 2DEG in strong magnetic fields

The $E(k)$ relation of a quasi two-dimensional electron gas is characterized by the equation

$$E(k) = E_j + E(k_x, k_y), \quad j = 0, 1, 2, \ldots, \tag{7}$$

where $E(k_x, k_y)$ is the kinetic energy of the electrons for the motion in the plane of the 2DEG (x–y plane) with the wavevector (k_x, k_y). The discrete subband energy E_j originates from the confinement of the electron gas within a narrow potential well in the z direction. It is well known that such a 2DEG can be formed closed to the semiconductor–insulator interface of a Metal–Oxide–Semiconductor Field-Effect Transistor [9] or at the interface between two semiconductors [10] (in our measurements (100)Si MOSFETs or GaAs–$Al_xGa_{1-x}As$ heterostructures). The energy separation between the electric subbands is typically 10 meV.

If a strong magnetic field B is applied in the z direction, the electrons will move on cyclotron orbits and the quasi continuous energy spectrum with the quantum numbers k_x and k_y becomes rearranged which leads to well separated energy levels E_n with $n = 0$, 1, 2, ... and a degeneracy of each level of $N = eB/h$. This degeneracy factor N corresponds to the number of cyclotron orbits per unit area and is independent of the semiconductor [11]. Since the Hall voltage of a 2DEG is given as

$$U_{\mathrm{H}} = BI/n_{\mathrm{inv}}e \tag{8}$$

(I = current, n_{inv} = carrier density per unit area), the Hall resistance $R_{\mathrm{H}} = U_{\mathrm{H}}/I$ under experimental conditions where an integer number i of Landau levels is occupied ($n_{\mathrm{inv}} = iN$) becomes

$$R_{\mathrm{H}} = h/e^2 i. \tag{9}$$

This equation forms the basis for the determination of h/e^2 and therefore of the fine structure constant.

Under the condition of fully occupied Landau levels the conductivity component σ_{xx} becomes zero, because the electrons cannot scatter elastically from an occupied state to an empty state, since these states are separated by an energy gap. Therefore the center of the cyclotron orbit cannot diffuse in the direction of the electric field. The conductivity component σ_{xx} can be measured directly with a circular sample (Corbino geometry), whereas for a long device with only one component for the current direction (Hall geometry) the resistivity components ρ_{xx} and ρ_{xy} are determined [12]. Typical device geometries used in our experiments are shown in fig. 1. Since the boundary conditions (equipotential lines for the highly doped source and drain constacts) may lead to corrections in ρ_{xx} and ρ_{xy} measurements, the experimental resistivity data are called R_x and R_H. However, calculations show [19,20] that for measurements of the quantized Hall resistance the corrections at the present

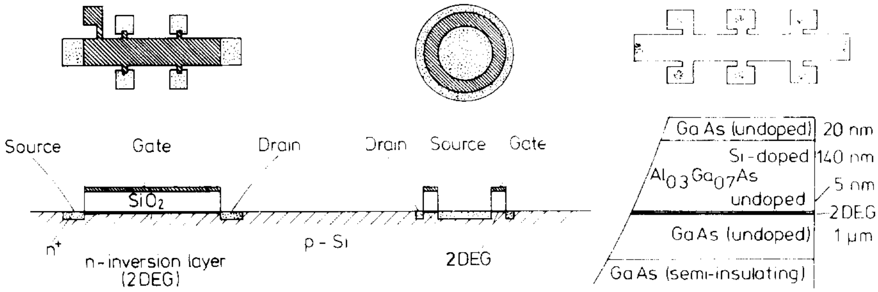

Fig. 1. Typical geometries and cross-sections of devices used in the experiments. From left to right: (a) Long silicon MOS device (Hall geometry) with potential probes (for R_x measurements) and Hall probes (for R_H measurements); typical length: 0.5 mm. (b) Circular MOS device for σ_{xx} measurements. (c) Cross-section and top view of a GaAs–$Al_{0.3}Ga_{0.7}As$ heterostructure with Hall geometry.

level of experimental uncertainty are unimportant and the measured resistivities R_x and R_H can be set equal to ρ_{xx} and ρ_{xy}.

The resistivity component ρ_{xx} is directly proportional to the conductivity component σ_{xx} [12] ($\sigma_{xx} = \sigma_{yy}$, $\sigma_{xy} = -\sigma_{yx}$)

$$\rho_{xx} = \sigma_{xx} / \left(\sigma_{xx}^2 + \sigma_{xy}^2 \right). \tag{10}$$

This means that the condition $\sigma_{xx} = 0$ (fully occupied Landau levels) leads to $\rho_{xx} = 0$ (no voltage drop between the potential probes). A correct value for the quantized Hall resistance $R_H = h/e^2 i$ is only expected under the condition $\rho_{xx} = 0$.

3. Experimental results

3.1. Measurements on silicon MOSFETs

Fig. 2 shows $R_x (\approx \rho_{xx})$ and $R_H (\approx \rho_{xy})$ data obtained from measurements on (100) silicon inversion layers. Since the carrier concentration n_{inv} of a 2DEG in a MOSFET and therefore the occupation of Landau levels can simply be varied by changing the gate voltage V_g, the curves in fig. 2 are plotted as a function of V_g at a fixed magnetic field of $B = 18.9$ T. The Hall resistance shows characteristic steps at values $R_H = h/e^2 i$, independent of the geometry of the device and the strength of the magnetic field [4,19]. Details of the experimental data obtained at gate voltages close to the occupation of the Landau level $n = 0$ (corresponding to four occupied levels $i = 4$ due to spin and valley degeneracy) are shown in fig. 3 [21]. The observed change $\Delta R_H / R_H$ in the plateau region is at least four orders of magnitude smaller than the change

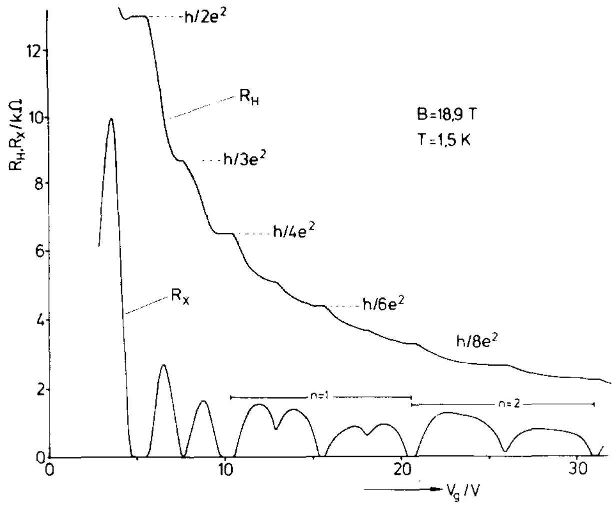

Fig. 2. Gate voltage dependence of the measured Hall resistance R_H and resistivity R_x at $B = 18.9$ T for a long silicon MOS device (length to width ratio $L/W = 8$, distance between potential probes $\Delta = L/3$).

in the gate voltage $\Delta V_g/V_g$. This result is incompatible with the usually accepted assumption that the carrier concentration n_{inv} of the 2DEG increases linearly with the gate voltage and that the Hall resistance at a fixed magnetic field decreases monotonically with n_{inv} like $R_H \sim 1/n_{inv}$. The observed steps in

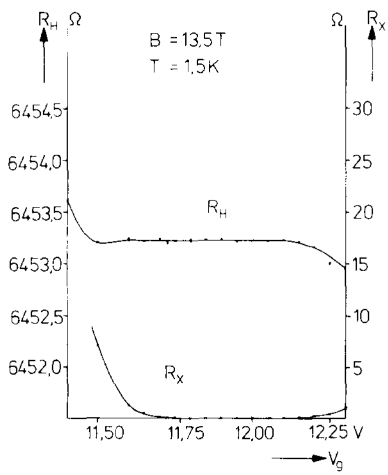

Fig. 3. High-resolution measurements of the Hall resistance R_H and the resistivity R_x for a silicon MOSFET at gate voltages close to the occupation of the $n = 0$ Landau levels (after ref. [21]).

$R_H(V_g)$ are related to "localized" regions in ρ_{xx}, and the following mechanisms can explain the experimental data.
(a) The carrier concentration n_{inv} in the inversion layer remains constant within the localized region, since at these gate voltages states not connected with the inversion layer become occupied (e.g. states in the depletion layer, in the SiO_2, etc.). This process seems to be important in GaAs–$Al_xGa_{1-x}As$ heterostructures [13].
(b) The carrier concentration in the inversion layer changes proportionally to the gate voltage but in the region of the steps in $\rho_{xy}(V_g)$, localized states in the tails of the Landau levels are occupied. Prange [14] as well as Aoki [15] calculated the contribution of localized states to the ρ_{xy} component and found that localized states do not contribute to ρ_{xx} but contribute to ρ_{xy} in such a way that a gate voltage-independent value corresponding to a fully occupied Landau level is expected.
(c) The possibility of electron–electron correlation effects (Wigner crystallization) has been discussed in connection with the phenomena of localiztion [16] ($\sigma_{xx} = 0$), but corresponding calculations for the Hall effect are not available.

Up to now it was not possible to decide which one of these mechanisms is responsible for the steps in the $\rho_{xy}(V_g)$ curves. All theoretical calculations [13–18]] published after the discovery of the quantized Hall resistance show that independent of the origin of the plateaus, the correct value $\rho_{xy} = h/e^2 i$ is expected as long as the condition $\rho_{xx} = 0$ is fulfilled.

Since ρ_{xx} is never exactly zero at finite temperatures, the most important correction to the Hall resistance seems to be the finite scattering time ($\rho_{xx} \neq 0$). However, our measurements show [21] that even an increase of ρ_{xx}^{min} by a factor of 10 leads to an unmeasurable correction to ρ_{xy}. If the correction is proportional to ρ_{xx}, we can estimate $\Delta\rho_{xy} < 0.1\rho_{xx}$. Therefore, the correction to R_H at the minimum of R_x in fig. 3 should be smaller than 10^{-7}. This correction is still smaller if, as assumed by Laughlin [17], ΔR_H changes with the square of the resistivity R_x.

3.2. Measurements on GaAs–$Al_xGa_{1-x}As$ heterostructures

Fig. 4 shows an experimental result for the resistivity $\rho_{xx}(B)$ and the Hall resistance $\rho_{xy}(B)$ of a GaAs–$Al_xGa_{1-x}As$ heterostructure with a carrier concentration of $n_{inv} \approx 5 \times 10^{11}$ cm^{-2}. Since the carrier concentration is fixed, the population of the Landau levels is changed by varying the magnetic field and therefore the degeneracy $N = eB/h$ of each level. At magnetic field values where ρ_{xx} goes to zero (fully occupied Landau levels), ρ_{xy} becomes magnetic-field independent with values equal to $h/e^2 i$ (i = number of filled levels). The origin of the plateaus in ρ_{xy} can be explained, if an exchange of electrons (tunneling) between the 2DEG and the donors in the $Al_xGa_{1-x}As$ is assumed. Baraff [13] calculated under this assumption self-consistently the ratio n_{inv}/B as a function of the magnetic field and found that this ratio remains constant

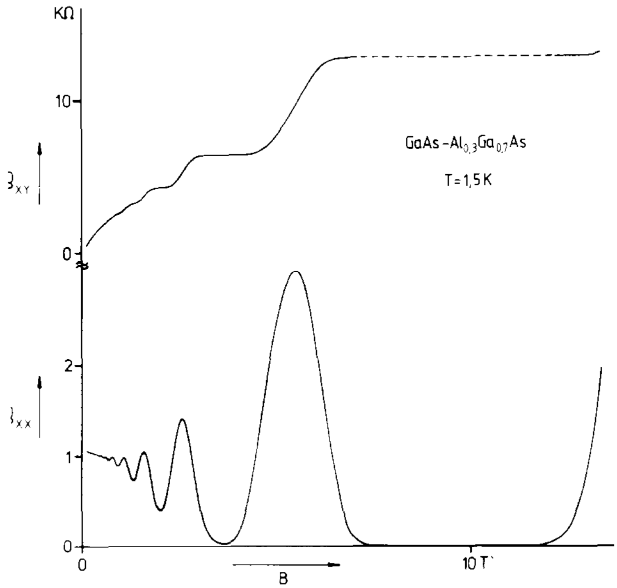

Fig. 4. ρ_{xy} and ρ_{xx} measurements on a GaAs–$Al_{0.3}Ga_{0.7}As$ heterostructure as a function of the transverse magnetic field. The inversion carrier concentration is about 5×10^{11} cm^{-2} [21]).

within certain magnetic field regions. This means that the plateaus in ρ_{xy} and ρ_{xx}, which look very similar to corresponding measurements on silicon MOSFETs, can be explained without assuming a localization of carriers in the tails of Landau levels.

The dotted part of the curves in fig. 4 characterizes the region where the relatively high "contact" resistance of the sample used in this experiment leads to an error in our potential measurements. For most of our silicon MOSFETs the contact resistance is negligibly small (< 100 Ω).

4. Conclusion

Measurements of the Hall resistance R_H of a two-dimensional electron gas, realized with a silicon MOS field effect transistor or a GaAs–$Al_xGa_{1-x}As$ heterostructure, demonstrate that under the condition of fully occupied Landau levels R_H is (within our uncertainty of about 10^{-6}) independent of external parameters. We believe that the value of R_H is given by the equation $R_H = h/e^2 i$ (i = number of fully occupied energy levels) as expected for an ideal two-dimensional electron gas, since all known corrections (finite scattering rate, interaction between Landau levels, shorting of the Hall voltage at the contacts) should change with magnetic field, temperature or sample geometry, which is not observed. Up to now, the finite scattering rate ($\sigma_{xx} \neq 0$) at $T \neq 0$ seems to be the limiting factor for high precision measurements of h/e^2. For the silicon MOSFETs used in our experiments ($B = 18$ T, $T = 1.5$ K) this correction should be of the order $\Delta\rho_{xy}/\rho_{xy} \approx 10^{-8}$. For high quality GaAs–$Al_xGa_{1-x}$As heterostructures, this correction is expected to be much smaller due to the high mobility ($\mu = 100{,}000$ $cm^2/V \cdot s$) and small effective mass ($m^* = 0.067 m_0$), which leads to a much larger energy gap between Landau levels and consequently to a lower σ_{xx}^{min} value than observed for silicon MOSFETs. At present no objections are known which could prevent the application of the quantized Hall resistance as a resistance standard with values based exclusively on the fundamental constant h/e^2.

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