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Physics and application of the quantum Hall effect

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Abstract

The High Magnetic Field Laboratory in Grenoble is the birthplace of the quantum Hall effect (QHE). In the morning of the 5th of February 1980 – during a magnetotransport experiment on silicon field-effect transistors – the idea came up to measure deviations of the Hall resistance relative to a quantized value which can be calculated on the basis of a simple one-electron picture. The analysis of the experimental data showed immediately that the deviations are unmeasurably small and today it is generally accepted that the quantized Hall resistance depends exclusively on fundamental constants. This result opened a new research field. In this paper only recent developments in the application of the QHE in metrology and the connection of the quantized Hall resistance with the quantization of the resistance of quantum point contacts, which was discovered in 1988, will be summarized.

1. Application in metrology

The quantum Hall effect (QHE) is well known for its application in metrology. Already the first publication of the QHE [1] with the original title "Realization of a Resistance Standard Based on Fundamental Constants" indicated that an application similar to the Josephson effect may be possible. Such an application has been recommended by the Comité Consultatif d'Electricité (CCE) in 1988 [2] and adopted by the Comité International des Poids et Mesures (CIPM) [3]. This recommendation says that starting from 1.1.1990 all calibrations of resistances will be based on the quantum Hall effect with a fixed value $R_{K-90} = 25812.807\ \Omega$ for the *von Klitzing constant* R_K which is identical with the fundamental value of the quantized Hall resistance (QHR). This value is the result of high precision measurements of the QHR at different national laboratories on the basis of the best resistance standards calibrated in SI units.

Since the QHR is more stable and more reproducible than any other resistor, one has fixed the value $R_{K\text{-}90}$ without any uncertainty in order to allow international comparisons of resistances with higher accuracy than realizable in SI units. (The index 90 for the constant R_K indicates that this is the best value in 1990.) As a consequence of this definition the $1\,\Omega$ resistors in the national laboratories changed their value at the beginning of 1990. For example, in the United States (NIST) and in Germany (PTB) the $1\,\Omega$ wire resistors, which formed the basis for resistance calibrations in these countries up to 1990, changed their value in the following way:

$$1\,\Omega_{\mathrm{NIST}} = 1\,\Omega - 1.690\,\mu\Omega - (0.0529\,\mu\Omega/a)\,t,$$

$$1\,\Omega_{\mathrm{PTB}} = 1\,\Omega - 0.56\,\mu\Omega - (0.013\,\mu\Omega/a)\,t,$$

$$t = 0 \quad \text{for } 1.1.1990.$$

$1\,\Omega$ denotes the unit of resistance deduced from the quantized Hall resistance. The stability and reproducibility of the quantized Hall resistance is clearly demonstrated in measurements by Delahaye et al. [4] and Jeckelmann et al. [5] as shown in Figs. 1 and 2.

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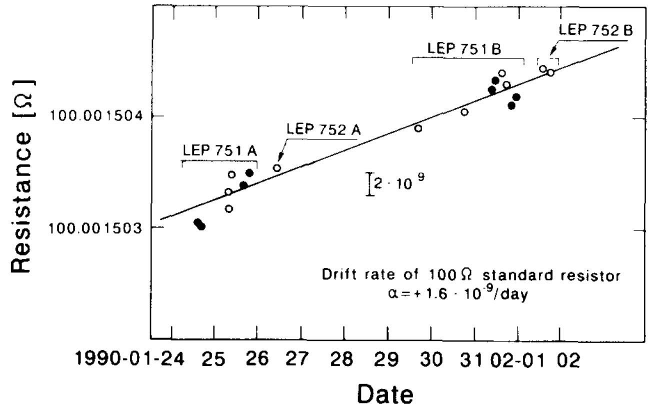

Fig. 1. Comparison of different quantized Hall resistances measured on four GaAs-based heterostructures. The 100 Ω standard resistor has an established drift rate of 1.6×10^{-9} per day. After Ref. [4].

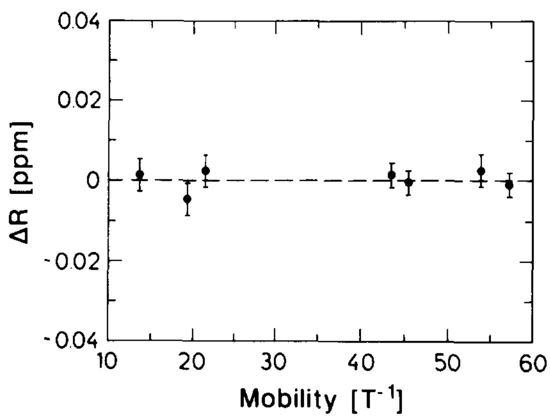

Fig. 2. Differences of the QHR measured in seven GaAs heterostructures. After Ref. [5].

In Fig. 1 the resistance of a 100 Ω wire resistor is measured as a function of time by using two different quantum Hall devices LEP 751 and LEP 752. The monotonic increase of the measured resistance is a result of the drift of the 100 Ω resistor with a rate of 1.6×10^{-9} per day. The measured curve is the same for the two different quantized Hall devices. This indicates that the QHR is very stable and independent of microscopic properties of the sample. Even large differences in the quality of the devices, characterized by different mobilities, do not influence the value of the quantized Hall resistance as long as the conditions described in the "Technical Guidelines for Reliable Measurements of the Quantized Hall Resistance" [6] are fulfilled. The results in Fig. 2 for seven different QHRs show that within the experimental uncertainty of $\pm 3.3 \times 10^{-9}$ a device independent value for the quantized Hall resistance is observed. A comparison between a silicon and a GaAs QHR showed that within an uncertainty of $\pm 3.5 \times 10^{-10}$ the result is even independent of the material [7].

A recent international comparison of the resistance standards of 10 countries confirmed that the worldwide uniformity in resistance calibrations has been improved by at least one order of magnitude due to the quantum Hall effect. The 1 Ω and 10 kΩ standard resistors of the national laboratories in Japan, Italy Russia, France, People's Republic of China, USA, United Kingdom, Canada, Switzerland, and Germany showed an excellent agreement [8]. The uncertainty of about 2×10^{-8} resulted mainly from the instabilities of the transportable resistors.

For a calibration of a resistor in SI units, one has to add a one-standard-deviation uncertainty of 2×10^{-7} so that the QHR does not have a fixed value $R_{K\text{-}90}$ but the value $R_K = (25812.807 \pm 0.005)\,\Omega_{SI}$. This error bar originates from the uncertainty in the realization of the unit of 1 Ω within the SI system by using the calculable Thomson–Lampard cross capacitor [9].

For the application of the QHE in metrology it is not necessary to identify the von Klitzing constant R_K with the fundamental constant h/e^2 as expected theoretically. However, for a QHE determination of the fine-structure constant α (which is exclusively a function of h/e^2) one needs the identity $R_K = h/e^2$.

A comparison of the fine-structure constant α determined by different methods is shown in Fig. 3 [10]. The reference line is based on the value $R_{K\text{-}90}$. In addition the following results are included: CODATA 1986 = recom-

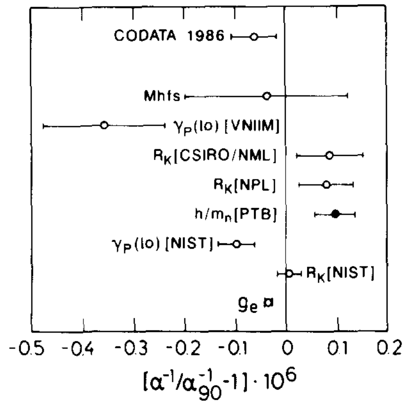

Fig. 3. Comparison of the inverse fine-structure constant α^{-1} determined by different methods (see text). After Ref. [10].

mended value for the fine-structure constant based on the last least-square adjustment of fundamental constants in 1986; Mhfs = Muonium hyperfine splitting; $\gamma_p(l_0)$ = gyromagnetic ratio of protons in low magnetic fields; h/m_n = determination of the fine-structure constant by measuring the velocity and wavelength of neutrons $v\lambda = h/m_n$; g_e = electron magnetic moment. The data for R_K are based on the assumption that the quantized Hall resistance is exactly identical with h/e^2.

Different approaches can be used to "prove" that the Hall resistance becomes quantized in units of h/e^2 under certain conditions and a large number of publications are available which discuss the QHE on the basis of the resistivity component ρ_{xy} of a two-dimensional electron gas in strong magnetic fields. The discrete energy spectrum (Landau levels) with a degeneracy corresponding to the number of flux quanta within the area of the sample can "explain" the QHR $R_K = h/e$ if exactly one energy level is fully occupied with electrons. If more than one Landau level is occupied (filling factor $i = 2, 3, \ldots$) the Hall resistance is reduced by this factor i. The gaps opened in the energy spectrum at fractional filling factor $i = \frac{1}{3}, \frac{1}{5}, \frac{2}{3}$, etc. lead to additional quantized Hall resistances but this fractional QHE, which is the result of electron–electron interaction, will not be discussed in this paper. Localized states stabilize the value of the quantized Hall resistance even if the magnetic field or the electron concentration is slightly changed. The reader is encouraged to read the corresponding review articles [11–19]. However, in these publications the model used to discuss the QHE is normally different from the real experimental situation. Especially, neither the finite size of the device which never leads to a real gap between the Landau levels, nor the metallic contacts (reservoirs) which are necessary for the electric measurements are included in a theoretical model.

In the following a new approach to discuss the QHE on the basis of one-dimensional edge channels (which will be the textbook explanation of the QHE in the future) will be presented. Even if this edge channel picture does not describe completely the real situation in QHE experiments (finite temperature, large Hall voltages) and even if the QHE can be discussed as a bulk phenomenon, it seems to be the most direct explanation of the QHE for samples where the current and potential probes are parts of the device.

2. Explanation of the quantized Hall resistance within the edge channel picture

From the experimental point of view it is obvious that under quantum Hall conditions the edge of a two-dimensional electron gas (= lateral boundary of the system where the two-dimensional carrier density drops to zero) is extremely important in transport measurements. An electric current can flow through the device only if both the source and drain contact are connected by a common edge. Fig. 4 summarizes the different situations where the source and drain contacts are either located at the same edge (quantized two-terminal resistance $R_{SD} = h/ie^2$) or are connected to different edges (Corbino geometry, Fig. 4(c)) or to the inner part of the 2DEG (Fig. 4(b)). The infinite two-terminal resistance $R_{SD} = \infty$ under the latter two conditions can be reduced to the quantized value $R_{SD} = h/ie^2$ if a part of the 2DEG is removed as shown in Fig. 4(d).

In order to avoid the influence of series resistances in high precision measurements of the quantized Hall resistance one measures the four-terminal resistance $R_{SD,HH}$ rather than the two-terminal resistance R_{SD}. The index HH means that the potential is measured between the probes H (see Fig. 4(a)) while the current source is connected to the contacts S and D. The potential probes must be in contact with the edge, otherwise an infinitely large impedance is observed. Electrochemical potentials are only well defined at edges which are connected to the external voltage source. All these experimental results can be well understood within the edge channel model for a two-dimensional electron gas in strong magnetic fields [20]. This model is directly connected to the electronic transport in an ideal one-dimensional system discussed by Landauer [21]. He pointed out that the resistance of a system comprising a reservior, an electron waveguide, and a reservoir can be related to an electron transmission probability through the conductor. The characteristic feature of such a one-dimensional channel is that the cancellation of the energy dispersion relation $E(k)$ in the

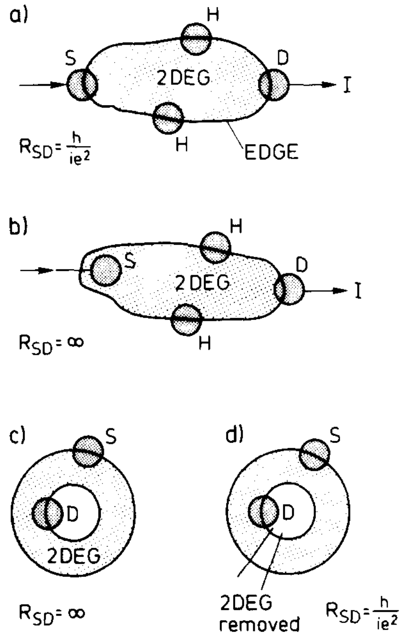

Fig. 4. Different arrangements of source (S) and drain (D) contacts which demonstrate the importance of the boundary of a two-dimensional electron gas (2DEG) in quantum Hall effect experiments: (a) Source, drain, and Hall contacts at the same edge (standard Hall geometry). Quantized two-terminal resistance. (b) Source contact not connected to the edge. A current I cannot flow in the quantum Hall regime. (c) Source and drain contact connected to *different* edges of the 2DEG (quasi-Corbino device). Infinitely large source–drain resistance. (d) The infinite source–drain resistance of a Corbino device is reduced to the quantized value h/ie^2 if a part of the 2DEG is removed in such a way that the contacts are connected by the same edge.

product of the density of states $D(E) = 1/\pi \mathrm{d}E/\mathrm{d}k$ and the group velocity $v = (1/(\mathrm{d}E/\hbar)\mathrm{d}k)$ leads to a quantized resistance $R_0 = h/2e^2$ (no backscattering, spin degeneracy included). Experimentally, such a quantization is observed in measurements on quantum point contacts. In these experiments the two-dimensional electron gas is depleted by means of a split-gate configuration which leads to a constriction in the 2DEG (point contact) with a width comparable to the Fermi wavelength [22, 23]. Under this condition one-dimensional subbands are formed. The number n of occupied one-dimensional electronic subbands (= channels) determines the value of the resistance across the point contact

$$R = \frac{h}{2e^2 n}. \tag{1}$$

This result for a point contact (which is obtained without a magnetic field B) is identical with the quantized Hall

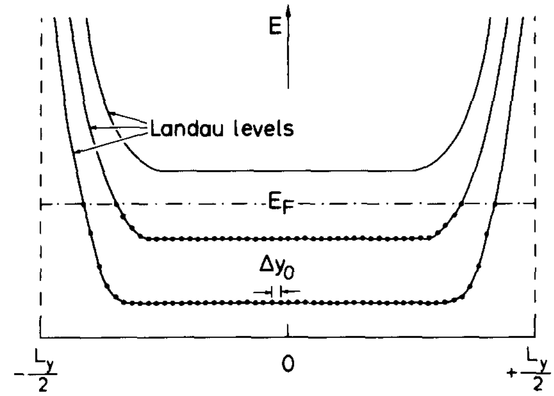

Fig. 5. Energy spectrum of ideal Landau levels for a sample with finite width L_y.

resistance if $2n$ is replaced by the filling factor $i = n_s h/eB$ (n_s = two-dimensional carrier density). The filling factor i corresponds to the number of spin-split energy levels of a two-dimensional system in strong magnetic fields whereas the spin degeneracy factor 2 is included in Eq. (1), since this degeneracy is not lifted without a magnetic field. The importance of the edge in QHE experiments and the connection between the one-dimensional ballistic transport and the QHE is clearly visible in the energy spectrum of a two-dimensional system in strong magnetic fields if the boundary is included. Fig. 5 shows the idealized energy spectrum of a noninteracting electron system without disorder in a confining potential. The width of the Hall device is characterized by the length L_y. Close to the boundary the Landau levels are shifted to higher energies. The dots correspond to the center coordinate y_0 of occupied electronic states for filling factor $i = 2$ (spin is not included in this picture). Since the center coordinates y_0 are directly connected to the k_x-vector of the plane wave part of the eigenfunction in the x-direction (= current direction), one can interpret Fig. 5 as the dispersion curve $E(k_x)$ of a one-dimensional system. The quantized resistance in one-dimensional quantum point contacts is only observable if backscattering $+k_x \rightarrow -k_x$ is suppressed. This corresponds to a vanishing scattering rate of electrons from one edge to the opposite edge in QHE experiments, equivalent to a vanishing conductivity σ_{xx} (Fermi energy within the mobility gap in the region between the edges). Since a finite group velocity $\mathrm{d}E/\mathrm{d}k_x$ is only present close to the edge (see Fig. 5), one can introduce one-dimensional edge currents corresponding to classical skipping orbits at the boundary of a conductor in strong perpendicular magnetic fields. The number N of one-dimensional edge channels is identical with the number of occupied

Landau levels which cross the Fermi energy close to the boundary of the sample. (It should be noted that the energy spectrum shown in Fig. 5 becomes more complicated if one includes screening effects. Under this condition the Landau levels are pinned at the Fermi energy since the electron concentration cannot change abruptly. This leads, at the edge, to compressible stripes of partly occupied Landau levels separated by incompressible stripes [24]. The situation becomes even more complicated when a current flows through the device since differences in the electrochemical potential at the edges results in a rearrangement of charges and in variations in the electric field. In addition, potential fluctuations with amplitudes larger than the cyclotron energy lead to crossings of the Fermi energy with the Landau levels not only at the boundary of the device but also in the inner part of the 2DEG. This leads to "inner edges" which do not contribute to an extra current since these inner edges form closed lines with a constant electrochemical potential and can be treated as localized states. These localized states pin the Fermi energy in the "gap" between the Landau levels which leads to the observed broadening of the quantized Hall plateaus.)

The discussion of the QHE within the Landauer–Büttiker formalism is as follows: If N energy levels (spin-split Landau levels) are fully occupied, N one-dimensional edge channels are present. The transport current flowing out of a reservoir j (metallic contact with chemical potential μ_j) is

$$L_{\text{out}}(j) = N\frac{e}{h}\mu_j. \quad (2)$$

The total current I_{tot} of a contact (= reservoir) is the difference in the currents carried by the outgoing and the incoming channels. For the following we choose the magnetic field direction such that the edge currents flow clockwise. The direction of the transport current depends on the chemical potentials and is independent of the direction of the edge current. For the simple situation shown in Fig. 6(a) with the filling factor $N = 1$ (only one edge channel) it is immediately clear (see Eq. (2)) that $\mu_1 = \mu_2 = \mu_3$ and $\mu_4 = \mu_5 = \mu_6$ with $\Delta\mu = \mu_1 - \mu_4 = (h/e)I$. This corresponds to the quantized Hall resistance $R_{14,26} = R_{14,35} = R_{14,14} = h/e^2$ and a vanishing longitudinal resistance $R_{14,23} = R_{14,65} = 0$.

The notation "edge current" means that in the ideal case the current flows close to the edge within the length scale of the magnetic length. This may be true for a two-dimensional electron gas confined by a wall potential. In reality the edge of a sample is produced by an etching process which forms an electrostatically defined confining potential with a characteristic depletion length of the order of 0.1 μm. Therefore the edge channels (= Landau levels crossing the Fermi energy close to the edge) are separated from the geometrical edge by the depletion length. An edge current can be defined as ideal when only the occupation of the Landau levels close to the edges is modified but the Landau level energies remain unchanged. The existence of edge currents can be demonstrated if *adjacent* edge channels (originating from different Landau levels) are occupied up to *different* electrochemical potentials. Under this condition the adjacent edge channel carry different electric currents which demonstrates that a current flow within a region corresponding to the spatial separation of adjacent edge channels is present. Such adiabatic transport is a direct proof for the existence of edge currents and some unexpected results in Hall effect experiment devices shown in Figs. 6(b) and (c) can be simply explained within the edge channel picture.

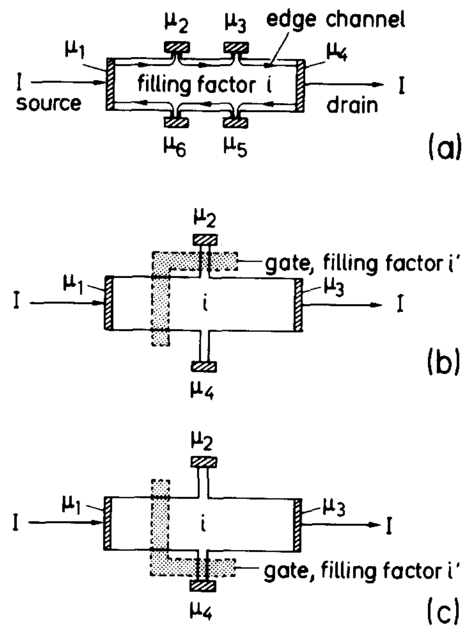

Fig. 6. (a) Sketch of a standard Hall device with chemical potentials μ_i for the current and potential contacts. (b) Hall device with a gate (dotted area) which allows a variation of the carrier density (filling factor i') across the sample and simultaneously across one of the potential arms. In the adiabatic regime the Hall resistance depends exclusively on the filling factor i' and not on the filling factor i of the main part of the device close to the Hall probes. (c) Hall device with a gate (dotted area) similar to Fig. 6(b). The measured Hall resistance is determined by the filling factor i (even in the adiabatic regime). This result is expected within the edge channel picture. When the magnetic field direction is reversed, the experimental results for the configurations shown in Figs. 6(b) and (c) are interchanged.

The situation shown in Figs. 6(b) and (c) seems to be equivalent to a Hall device with filling factor i except that a gate (dotted area) produces in the two regions which overlap with the two-dimensional electron gas a smaller carrier density with a smaller filling factor $i' < i$. These additional resistances correspond to a series resistance in the current path and an additional resistance in the leads for the Hall potential measurements which should (in the classical picture) not influence the Hall resistance $R_{13,24} = h/ie^2$ as long as a constant current I is maintained and the voltmeter has a sufficient large input impedance. However, the edge channel picture with adiabatic transport predicts that only for the situation shown in Fig. 6(c) the expected Hall resistance with the filling factor i can be observed whereas the situation shown in Fig. 6(b) leads to a Hall resistance corresponding to the filling factor i' of the dotted part of the device. These results are observed experimentally [25] as long as in Fig. 6(c) the distance along the boundary of the sample between the gate barrier across the current path and the gate barrier across the potential arm is smaller than the equilibration length for interedge channel scattering which is typically 100 μm at low temperatures. Up to now, only the edge channel picture can explain the experimental results in the adiabatic regime so that the edge current picture is generally accepted for a discussion of the quantum Hall effect. On the other hand it should be noted that the quantized Hall resistance is insensitive to the current distribution within the two-dimensional system and that for large current densities the QHE cannot be simply described by edge currents located within the depletion length at the boundary. The current penetrates into the interior of the two-dimensional system with increasing Hall voltage or increasing temperature and behaves more and more like a bulk current. Measurements on Corbino-like devices have demonstrated that a quantized Hall conductivity can be observed even without an edge [26]. This means that the QHE is basically not an edge effect, but for experiments on Hall devices and small device currents $I_{tot} < 0.1$ μA the current flow close to the edge dominates the electronic properties. In this case, the edge channel picture is a realistic description of the quantum Hall effect.

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